9-1 day 3 Direct Comparison & Limit Comparison Test

Learning Objectives:

I can use the direct comparison test to determine whether an infinite series converges or diverges

I can use the limit comparison test to determine whether an infinite series converges or diverges

The Direct Comparison Test

If $a_n \le b_n$ for all $n \ge N$ for some N and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges too.

Similarly, if $a_n \le b_n$ for all $n \ge N$ for some N and $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges too.

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Ex1. Use the Direct Comparison Test to determine if each series converges.

1.) $\sum_{n=1}^{\infty} \frac{1}{n^3+1} < \sum_{n=1}^{\infty} \frac{1}{n^3}$

✓ Converges D.C.T.

Converge p-test $p>1$

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2.) $\sum_{n=1}^{\infty} \frac{1}{\ln n} > \sum_{n=1}^{\infty} \frac{1}{n}$

diverges
D.C.T.

diverges
p-test
p=1

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3.) $\sum_{n=1}^{\infty} \frac{1}{2+3^n} < \sum_{n=1}^{\infty} \frac{1}{3^n}$

$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$

Geo

$r < 1$

converges

converges

D.C.T.

4.) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} \cdot 2^n} < \sum_{n=1}^{\infty} \frac{1}{2^n}$

geo
$r = \frac{1}{2} < 1$
converge

converge
D.C.T.

The Limit Comparison Test

Suppose that a_n and b_n are positive

$L \neq 0$

$L \neq \infty, -\infty$

sequences and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$

Where L is finite and positive, then either BOTH series converge or BOTH series diverge.

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Ex2. Use the Direct Comparison Test to determine if each series converges.

1.) $\sum_{n=1}^{\infty} \frac{1}{2^n - 1} \Longleftrightarrow \sum_{n=1}^{\infty} \frac{1}{2^n}$ converge geo $r < 1$

$\lim_{n\to\infty} \frac{\frac{1}{2^n}}{\frac{1}{2^n-1}} = \lim_{n\to\infty} \frac{2^n-1}{2^n}$

$= \lim_{n\to\infty} 1 - \frac{1}{2^n} = 1$

✓ converges too L.C.T.

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2) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1} \longrightarrow$ converge

$$\frac{\sqrt{n}}{n^2+1} \Rightarrow \frac{\sqrt{n}}{n^2} = \frac{n^{1/2}}{n^2} = \frac{1}{n^{3/2}}$$

converge
p-test
$p>1$

$$\lim_{n\to\infty} \frac{\frac{1}{n^{3/2}}}{\frac{\sqrt{n}}{n^2+1}} = \lim_{n\to\infty} \frac{1}{n^{3/2}} \cdot \frac{n^2+1}{n^{1/2}}$$

$$= \lim_{n\to\infty} \frac{n^2+1}{n^2} = \lim_{n\to\infty} 1 + \frac{1}{n^2} = \boxed{1}$$

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Homework

Direct Comparison and Limit Comparison worksheet

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